

THEORETICAL ESTIMATE(S) OF THE CP-VIOLATING QUANTITY ε'/ε IN $K \rightarrow 2\pi$ DECAYS

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Abstract.

I give a short presentation of the theoretical prediction of ε'/ε . Short distance and especially long distance aspects of the computation is discussed. I concentrate on the general framework and the chiral quark model approach, while other approaches are also shortly presented. Because of the intrinsic uncertainties of the long-distance computations, it is unlikely that new physics effects can be disentangled from the standard model prediction.

1. Introduction

The physical neutral kaons K_S and K_L decay predominantly into two and three pions, respectively. The final states of two and three pions are even and odd under a CP transformation, respectively. Therefore, it was natural to think of $K_{S,L}$ as eigenstates of CP -symmetry. But in 1964 the CP -violating decay mode $K_L \rightarrow 2\pi$ was discovered. This shows that $K_{S,L}$ are linear combinations of the CP -parity eigenstates $\sim (K^0 \pm \bar{K}^0)$:

$$K_{S,L} \simeq \left[(1 + \varepsilon)K^0 \pm (1 - \varepsilon)\bar{K}^0 \right] / \sqrt{2} ; \quad (1)$$

where $|\varepsilon| = (2.266 \pm 0.017) \times 10^{-3}$.

While ε is determined by CP -violation in the $\Delta S = 2$ transition $K^0 \leftrightarrow \bar{K}^0$, CP -violation can also proceed *directly* in the decays of K^0 or \bar{K}^0 in two pions (the ε' effect), expressed by the parameters:

$$\eta_{ab} \equiv \frac{\langle \pi^a \pi^b | \mathcal{L}_W | K_L \rangle}{\langle \pi^a \pi^b | \mathcal{L}_W | K_S \rangle} ; \quad \eta_{00} \simeq \varepsilon - 2\varepsilon' , \quad \eta_{+-} \simeq \varepsilon + \varepsilon' , \quad (2)$$

where \mathcal{L}_W is the $\Delta S = 1$ weak lagrangian.

In the Standard Model (SM), ε and ε' are proportional to the same complex quantity in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. In models beyond the SM there might be different sources of CP -violation. It is therefore of great importance to establish the experimental value of ε' and discuss its theoretical value within the SM and beyond.

In this presentation, I will concentrate on qualitative aspects. For technical details I refer to reviews [1, 2, 3, 4] and recent papers [5, 6] which also contains the most important references on the subject.

2. Effective Theory for $\Delta S = 1$ Non-leptonic Decays

Theoretically, the amplitudes for $K \rightarrow 2\pi$ depend on different energy scales from the top quark mass down to the masses of the light u, d -quarks. In such cases, one often uses *effective theories* [7]. In this case one constructs an effective quark level Lagrangian

$$\mathcal{L}_W = \sum_i C_i(\mu) Q_i(\mu) , \quad (3)$$

where all information of the short distance (SD) loop effects above a renormalization scale μ is contained in some (Wilson) coefficients C_i . These depend on the masses of the W, Z -bosons, the heavy quark masses ($m_q > \mu$) and moreover on Λ_{QCD} and the renormalization scheme. The Q_i 's are quark operators, typically containing products of two quark currents. A priori, a huge number of operators are involved, depending on the chosen μ .

To obtain a physical amplitude, one has to calculate the hadronic matrix elements of the quark operators Q_i within some non-perturbative framework. This is the long distance (LD) part of the calculation. Unfortunately, lattice gauge theory results are difficult to obtain for some operators. Therefore models and assumptions are often used. Using quark models, μ is usually taken below the charm quark mass. Then the Q_i 's contain only the lightest three quarks u, d, s , and the dominating operators are considered to be ten operators of dimension 6 (see, however, [8]).

For the low-energy sector containing the three light quarks only, there is a well defined effective theory, chiral perturbation theory (χPT), having the basic symmetries of QCD. One can try to match these theories by *bosonizing* the quark operators Q_i :

$$Q_i \rightarrow \sum_j F_{ij} \hat{\mathcal{L}}_j , \quad (4)$$

where F_{ij} are quantities which have to be calculated with non-perturbative methods (including quark models). The $\hat{\mathcal{L}}_j$'s are chiral lagrangian terms,

(involving π, K, η), to be discussed in the subsections 2-4. Knowing the bosonization in (4), we could calculate the various $K \rightarrow 2\pi, 3\pi$ amplitudes from a $\Delta S = 1$ chiral lagrangian

$$\mathcal{L}_W(\chi PT) = \sum_j G_j \hat{\mathcal{L}}_j ; \quad G_j = \sum_i C_i F_{ij} . \quad (5)$$

The challenge within such an approach is that the coefficients should be calculated (and matched) in a region where both the SD and LD calculations are valid. Notice that in pure chiral perturbation theory the $\hat{\mathcal{L}}_j$'s are known from symmetry requirements, while the coefficients G_j has to be determined from experiment.

A starting point in many calculations has been the *factorization* hypothesis in cases where the quark operators contain a product of two currents, say. This hypothesis (also named “vacuum saturation approximation” (VSA)) is easy to handle because the matrix elements of currents are known. But corrections to VSA are in general large.

2.1. THE QUARK EFFECTIVE LAGRANGIAN AND THE WILSON COEFFICIENTS

The $\Delta S = 1$ quark effective lagrangian at a scale $\mu < m_c$ can be written as in (3) with

$$C_i(\mu) = -\frac{G_F}{\sqrt{2}} [\lambda_u z_i(\mu) + \lambda_t y_i(\mu)] . \quad (6)$$

Here G_F is the Fermi coupling, the functions $z_i(\mu)$ and $y_i(\mu)$ are the CP -conserving and -violating parts of the coefficients, and $\lambda_q = V_{qd} V_{qs}^*$ (for $q = u, t$) are the CKM factors. The numerical values of z_i and y_i are of order one down to 10^{-4} , and can be found in the litterature. The standard basis (for $\mu < m_c$) includes ten operators. We display four, which are important for the $\Delta I = 1/2$ rule ($Q_{1,2,6}$) and ε'/ε ($Q_{6,8}$):

$$\begin{aligned} Q_1 &= 4 (\bar{s}_L \gamma^\mu d_L) (\bar{u}_L \gamma_\mu u_L) , & Q_6 &= -8 \sum_q (\bar{s}_L q_R) (\bar{q}_R d_L) , \\ Q_2 &= 4 (\bar{s}_L \gamma^\mu u_L) (\bar{u}_L \gamma_\mu d_L) , & Q_8 &= -12 \sum_q \hat{e}_q (\bar{s}_L q_R) (\bar{q}_R d_L) , \end{aligned} \quad (7)$$

where \hat{e}_q are the quark charges ($\hat{e}_u = 2/3$, $\hat{e}_d = \hat{e}_s = -1/3$), and $q_{L,R}$ are the left- and right-handed projections of the quark fields. Within this basis the Wilson coefficients are calculated to the order α_s^2 and $\alpha_s \alpha_{em}$ [9, 10], and it is now a basic element used by all groups estimating ε'/ε .

Within the SM, Q_2 is induced by W -exchange at the tree level (see Fig. 1, left). Switching on QCD, Q_1 is induced by one loop corrections to Q_2 . The rest of the operators are induced by the penguin diagrams (see Fig. 1,

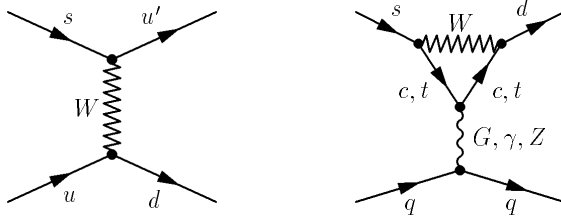


Figure 1. W -exchange corresponding to Q_2 , and Penguin diagram(s)

right) and their higher order loop corrections, Q_{3-6} by gluon (G) exchange and Q_{7-10} by photon (γ) or Z exchange (also box diagrams with exchange of two W 's enter).

Other possible operators of dimension 5 or 6 exist, but give small contributions within the SM [11, 12]. However, it has been stated that higher dimensional operators will become relevant[13] if the renormalization (separation) scale μ is chosen to low[8].

The $\Delta S = 2$ case (involving one relevant quark operator only) is treated along similar lines. Note that an analysis of the $\Delta S = 2$ K - \bar{K} mixing determines the CP-violating parameter $\text{Im } \lambda_t$ in the SM.

2.2. CHIRAL PERTURBATION THEORY (χPT)

The chiral lagrangian and chiral perturbation theory provide a faithful representation of the light (u, d, s quark) sector of the SM after the quark and gluon degrees of freedom have been integrated out. The form of this effective field theory and all its possible terms are determined by $SU_L(3) \times SU_R(3)$ chiral invariance and Lorentz invariance. Terms which explicitly break chiral invariance are introduced in terms of the quark mass matrix $\mathcal{M} = \text{diag}[m_u, m_d, m_s] = \mathcal{M}^\dagger$. The lowest order strong chiral lagrangian is

$$\mathcal{L}_{\text{strong}}^{(2)} = \frac{f^2}{4} \text{Tr} \left(D_\mu \Sigma D^\mu \Sigma^\dagger \right) + \frac{f^2}{2} B_0 \text{Tr} \left(\mathcal{M} \Sigma^\dagger + \Sigma \mathcal{M}^\dagger \right). \quad (8)$$

The parameter B_0 is defined by $\langle \bar{q}_i q_j \rangle = -f^2 B_0 \delta_{ij}$, where

$$(m_s + m_d) \langle \bar{q} q \rangle = -f_K^2 m_K^2, \quad (m_u + m_d) \langle \bar{q} q \rangle = -f_\pi^2 m_\pi^2, \quad (9)$$

in the PCAC limit. The $SU_L(3) \times SU_R(3)$ field

$$\Sigma \equiv \exp \left(\frac{2i}{f} \Pi(x) \right) \quad (10)$$

contains the pseudoscalar octet (the would-be Goldstone bosons π , K , η) $\Pi(x) = \sum_a \lambda^a \pi^a(x)/2$, ($a = 1, \dots, 8$). The coupling f is, to lowest order, identified with the pion decay constant f_π (and equal to f_K before chiral loops are introduced); it defines a characteristic scale $\Lambda_\chi \equiv 2\pi\sqrt{6/N_c} f \simeq 0.8$ GeV, at which chiral symmetry breaks down. When the matrix Σ is expanded in powers of f^{-1} , the lowest order term is the free Klein-Gordon lagrangian for the pseudoscalar particles.

To next-to-leading order there are ten terms and with coefficients L_i to be determined experimentally or by means of some model. Some of them play an important role in the physics of ε'/ε . As an example, we display the L_5 and L_8 terms in $\mathcal{L}_{\text{strong}}^{(4)}$ governing much of the penguin physics:

$$\begin{aligned} \mathcal{L}_{\text{strong}}^{(4)} = & \dots + L_5 B_0 \text{Tr} \left[D_\mu \Sigma D^\mu \Sigma^\dagger (\mathcal{M} \Sigma^\dagger + \Sigma \mathcal{M}^\dagger) \right] + \dots \\ & + L_8 B_0 \text{Tr} \left[\mathcal{M}^\dagger \Sigma \mathcal{M}^\dagger \Sigma + \mathcal{M} \Sigma^\dagger \mathcal{M} \Sigma^\dagger \right] + \dots \end{aligned} \quad (11)$$

2.3. BOSONIZATION OF CURRENTS AND DENSITIES

Using factorization (VSA) of currents and densities in the four quark operators Q_2 and Q_6 (as typical examples), one obtains

$$\langle \pi^+ \pi^- | Q_2 | K^0 \rangle = - \langle \pi^- | \bar{u} \gamma^\mu \gamma_5 d | 0 \rangle \langle \pi^+ | \bar{s} \gamma_\mu u | K^0 \rangle, \quad (12)$$

$$\begin{aligned} \langle \pi^+ \pi^- | Q_6 | K^0 \rangle = & 2 \langle \pi^- | \bar{u} \gamma_5 d | 0 \rangle \langle \pi^+ | \bar{s} u | K^0 \rangle - 2 \langle \pi^+ \pi^- | \bar{d} d | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | K^0 \rangle \\ & + 2 \left[\langle 0 | \bar{s} s | 0 \rangle - \langle 0 | \bar{d} d | 0 \rangle \right] \langle \pi^+ \pi^- | \bar{s} \gamma_5 d | K^0 \rangle, \end{aligned} \quad (13)$$

where the matrix elements of the densities $\bar{s} \gamma_5 u$ and $\bar{s} d$ might be obtained from PCAC from matrix elements of (vector and axial) currents. Let us stress that there is no theoretical underpinning for the VSA; it is just a convenient reference point which is equivalent to retaining terms of $O(1/N_c)$ in the $1/N_c$ -expansion, N_c being the number of colours.

From (8) and (11) we find the bosonized currents and densities:

$$\bar{q}_L^j \gamma^\mu q_L^i \rightarrow -i \frac{f^2}{2} \left[\Sigma^\dagger D^\mu \Sigma \right]^{ij}, \quad (14)$$

$$\bar{q}_L^j q_R^i \rightarrow -2B_0 \left[\frac{f^2}{4} \Sigma + L_5 \Sigma D^\mu \Sigma^\dagger D^\mu \Sigma + 2L_8 \Sigma \mathcal{M}^\dagger \Sigma \right]^{ij} \quad (15)$$

where i, j are flavour indices (u, d, s).

Using Fierz transformations and properties of the $SU(3)_c$ colour matrices T^a , the operator Q_1 (and similar for other operators) can be rewritten

$$Q_1 = \frac{1}{N_c} Q_2 + 8 (\bar{s}_L \gamma^\mu T^a u_L) (\bar{u}_L T^a \gamma_\mu d_L). \quad (16)$$

The last term in (16) corresponds to colour exchange between two currents and is genuinely non-factorizable.

2.4. THE WEAK CHIRAL LAGRANGIAN

The $\Delta S = 1$ chiral lagrangian at $O(p^2)$ might be written as nine terms, of which we explicitly display four:

$$\begin{aligned} \mathcal{L}_{\Delta S=1}^{(2)} = & G_{LR}^{(0)}(Q_{7,8}) \text{Tr} \left(\lambda_2^3 \Sigma^\dagger \lambda_1^1 \Sigma \right) + G_{\underline{8}}(Q_{3-10}) \text{Tr} \left(\lambda_2^3 D_\mu \Sigma^\dagger D^\mu \Sigma \right) \\ & + G_{LR}^{(m)}(Q_{7,8}) \left[\text{Tr} \left(\lambda_2^3 \Sigma^\dagger \lambda_1^1 \Sigma \mathcal{M}^\dagger \Sigma \right) + \text{Tr} \left(\lambda_1^1 \Sigma \lambda_2^3 \Sigma^\dagger \mathcal{M} \Sigma^\dagger \right) \right] \\ & + G_{LL}^a(Q_{1,2,9,10}) \text{Tr} \left(\lambda_1^3 \Sigma^\dagger D_\mu \Sigma \right) \text{Tr} \left(\lambda_2^1 \Sigma^\dagger D^\mu \Sigma \right) + \dots, \end{aligned} \quad (17)$$

where λ_j^i are combinations of Gell-Mann $SU(3)$ flavour matrices defined by $(\lambda_j^i)_{lk} = \delta_{il} \delta_{jk}$ and Σ is defined in eq. (10). In (17) we have indicated the quark operators from which the G_j 's get their contribution.

In the leading order in $1/N_c$ the two most important contributions to ε'/ε might be found by using (3, 14, 15).

$$G_{\underline{8}}(Q_6) = -24 \frac{\langle \bar{q}q \rangle^2 L_5}{f^2} C_6, \quad G_{LR}^{(0)}(Q_8) = -3 \langle \bar{q}q \rangle^2 C_8. \quad (18)$$

3. Theoretical Predictions

The generic amplitude for K^0 to decay into two pions is

$$\langle (\pi\pi)_{(I)} | \mathcal{L}_W | K^0 \rangle = -i A_I \exp(i \delta_I), \quad (19)$$

where the phases δ_I come from the pion final-state interactions (FSI). The decay of \bar{K}^0 is given by the same expression, except that A_I is replaced by its complex conjugate A_I^* . The phase is the same according to Watson's theorem. The smallness of $\omega \equiv (\text{Re } A_2 / \text{Re } A_0) \simeq 1/22.2$ is the $\Delta I = 1/2$ rule of $K \rightarrow \pi\pi$ decays. Any approach used to predict ε'/ε should also reproduce this important selection rule. ε'/ε can be written

$$\frac{\varepsilon'}{\varepsilon} = \frac{G_F}{2|\epsilon| \text{Re } A_0} \text{Im } \lambda_t [\omega \Pi_0 - \Pi_2], \quad (20)$$

where, referring to the $\Delta S = 1$ quark lagrangian of eq. (3),

$$\Pi_I = \frac{1}{\cos \delta_I} \sum_i y_i \text{Re} \langle (\pi\pi)_{(I=0,2)} | Q_i | K^0 \rangle \quad (21)$$

for isospin $I = 0, 2$ for the pions. For Π_0 , there is also an isospin breaking factor $(1 - \Omega_{IB})$ attached to the right hand side of (21). The value 0.25 was used for Ω_{IB} . Recent analysis give shifted values (see refs. 33-36 in [6]).

Notice that $\omega \Pi_0$ (dominated by Q_6) has the same sign and is, within the VSA, almost of the same order of magnitude as Π_2 (dominated by Q_8). Still, Q_6 gives the biggest contribution to ε'/ε . For instance, the Q_6 contribution is enhanced by chiral loops.

Notice also the explicit presence of the final-state-interaction phases in eq. (21). Their presence is a consequence of writing the absolute values of the amplitudes in term of their dispersive parts. A fit from the experimental data gives $\delta_0 \simeq 37^\circ$ and $\delta_2 \simeq -7^\circ$, in agreement with χPT . FSI therefore enhances the $I = 0$ over the $I = 2$ amplitude by about 20%. (This effect is not explicitly included in some existing estimates.)

3.1. THE CHIRAL QUARK MODEL (χQM)

This model has the following term added to the QCD lagrangian [14]:

$$\mathcal{L}_{\chi QM} = -M \left(\bar{q}_R \Sigma q_L + \bar{q}_L \Sigma^\dagger q_R \right), \quad (22)$$

which introduces meson-quark couplings. The quantity M is interpreted as the constituent quark mass, appearing because of chiral symmetry breaking.

Within the χQM , the matrix elements in (12, 13) are evaluated to $O(p^4)$ within the model. The current matrix elements of lowest order are well known, and the χQM version of these (given by divergent quark loops), are in agreement with these by construction. Furthermore, the L_i 's of (11) can be calculated, for instance

$$L_5 = -\frac{f^4}{8\langle \bar{q}q \rangle} \frac{1}{M} \left(1 - 6 \frac{M^2}{\Lambda_\chi^2} \right). \quad (23)$$

The model has a “rotated picture”, where the term $\mathcal{L}_{\chi QM}$ in (22) is transformed into a pure mass term $-M\bar{\chi}\chi$ for rotated “constituent quark fields” $\chi_L = \xi q_L$ and $\chi_R = \xi^\dagger q_R$, where $\xi \cdot \xi = \Sigma$. The meson-quark couplings in this rotated picture arise from the kinetic part of the constituent quark lagrangian. These interactions can be described in terms of vector and axial vector fields coupled to the constituent quark fields. The axial field, being invariant under local chiral transformations, is given by

$$2i\mathcal{A}_\mu = \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger = -\xi^\dagger (D^\mu \Sigma) \xi^\dagger = \xi (D_\mu \Sigma^\dagger) \xi. \quad (24)$$

Using (24), the strong chiral lagrangian $O(p^2)$ can be understood as two axial fields \mathcal{A}_μ coupled to a quark loop, giving $\mathcal{L}_s^{(2)} \sim \text{Tr} [\mathcal{A}_\mu \mathcal{A}^\mu]$. The QCD current mass lagrangian can be transformed to the form

$$\mathcal{L}_{cm} = -\bar{\chi} \left(\widetilde{M}_q^V + \gamma_5 \widetilde{M}_q^A \right) \chi, \quad \widetilde{M}_q^{V,A} \equiv \frac{1}{2} \left(\xi^\dagger \mathcal{M}_q \xi^\dagger \pm \xi \mathcal{M}_q^\dagger \xi \right). \quad (25)$$

Similarly, a lefthanded current can be written (λ^X is a SU(3) flavour matrix)

$$\bar{q}_L \gamma^\mu \lambda^X q_L = \bar{\chi}_L \gamma^\mu \Lambda^X \chi_L ; \quad \Lambda^X \equiv \xi \lambda^X \xi^\dagger \quad (26)$$

By coupling the fields \mathcal{A}_μ , $\widetilde{M}_q^{V,A}$, Λ^X to quark loops, the chiral lagrangians in sect. 2.2-4, and higher order terms can be calculated.

Within the χQM one can include soft gluon contributions corresponding to the nonfactorizable term in (16). One calculates soft gluon emission from the two coloured currents, and identify the product of two gluon field tensors with the gluon condensate $\langle G^2 \rangle$. As an example [15, 16]:

$$G_{LL}^a(Q_1) = \frac{1}{N_c} f_\pi^4 (1 - \delta_G) \quad , \quad \delta_G \equiv N_c \frac{\langle \alpha_s G G / \pi \rangle}{32\pi^2 f^4} . \quad (27)$$

The hadronic matrix elements of Q_i in (21) to $O(p^4)$ has three terms:

- i) A three level term obtained from the $O(p^2)$ chiral lagrangian (17).
- ii) The chiral loops contributions from (17), being $O(p^4)$.
- iii) $O(p^4)$ terms obtained by calculating the matrix elements in (12,13) up to $O(p^4)$ (as we did in [16, 17, 18, 6]), or equivalently, one may calculate the G_i 's of the $O(p^4)$ weak lagrangian.

The hadronic matrix elements have a scale dependence from the chiral loops which is matched to the one in the Wilson coefficients at the scale $\mu = \Lambda_\chi \simeq 0.8$ GeV. We find [17] that the $\Delta I = 1/2$ rule is well reproduced for the reasonable values (see also [15]. Notice that $\delta_G \simeq 2$.)

$$M \simeq 200 \text{ MeV} , \quad \langle \alpha_s G G / \pi \rangle \simeq (330 \text{ MeV})^4 , \quad \langle \bar{q}q \rangle \simeq (-240 \text{ MeV})^3 . \quad (28)$$

These are used as input to estimate ε'/ε , and we got [18] a result close to the one which later became the world average $\text{Re } \varepsilon'/\varepsilon = (19 \pm 2.4) \times 10^{-4}$, based on results from 1992 at CERN and FNAL, and the last years run at FNAL (KTeV) [19] and CERN (NA48) [20]. Our ("Trieste group") estimate is shown together with other theoretical estimates in Fig. 2. For precise statements about numerical values, I refer to the cited papers.

The strength of our approach [16, 17, 18] is that all contributions are calculated systematically up to $O(p^4)$ in the chiral expansion. The weakness is its model dependence, which it shares more or less with other estimates. A matching of SD and LD calculations at $\mu \simeq 0.8$ GeV has been questioned. Still, the obtained numerical stability in the matching is good.

3.2. OTHER APPROACHES

- München[5]: The $\Delta I = 1/2$ rule is fitted (at $\mu = m_c$). Unfortunately, this does not give (enough) information about the matrix elements of $Q_{6,8}$. These are varied around their leading $1/N_c$ values.

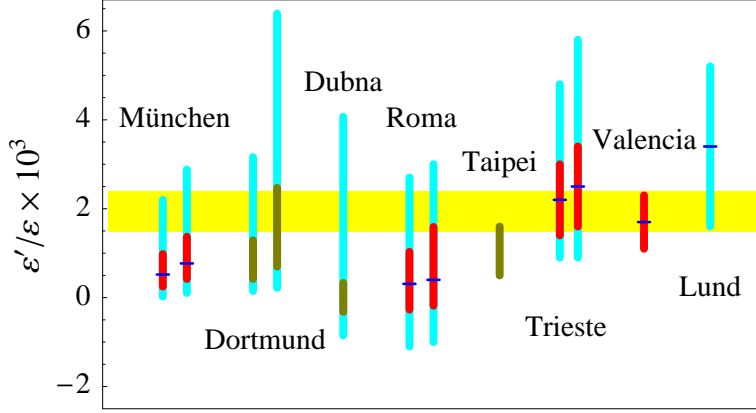


Figure 2. Summary of theoretical estimates for ε'/ε . The dark (light) range corresponds to Gaussian (flat) scanning of the uncertainties. The gray horizontal band is the average experimental result.

- Roma[21]: This is the only first principle estimate. Some matrix elements have been calculated, but unfortunately not that of Q_6 , which is varied by 100% around its VSA value. Also the “penguin contractions” of $Q_{1,2}$ needed to explain the $\Delta I = 1/2$ rule are difficult to obtain.
- Dortmund [22]: The quark operators are matched (between 600 and 900 MeV) directly to their corresponding chiral loops with a quadratic cut-off, carefully identified with the renormalization scale.
- Dubna[23]: Uses extended Nambu Jona-Lasinio (ENJL) models and chiral loops regularized by the heat kernel. It is found that chiral loops to $O(p^6)$ have sizeable absorptive parts but rather small real parts.
- Taipei[24]: Uses “generalized factorization” where scheme dependence is absorbed in effective Wilson coefficients. Non-factorizable contributions in $(16,27)$ are represented by a phenomenological parameter.
- Valencia[25]: Special attention is put on FSI effects. A dispersion relation is used, and the analysis confirms the importance of chiral loops.
- Lund[26]: A EJNL framework including axial and vector resonances. Care is taken to match LD and SD calculations. A sizeable enhancement of the Q_6 contribution is found, which predicts ε'/ε on the high side.

Also, an estimate using a linear sigma model [27] shows enhancement of the Q_6 matrix element, but it's hard to explain the $\Delta I = 1/2$ rule and ε'/ε at the same time within this approach.

4. Conclusions

All groups have to a certain extent used assumptions and models. Still, most estimates are roughly in the right ballpark, as seen from Fig. 2. Given the hadronic uncertainties, it will be hard to disentangle new physics from the SM predictions.

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